NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3223

AN ANALYSIS OF SHOCK-WAVE CANCELLATION AND REFLECTION FOR POROUS WALLS WHICH OBEY AN EXPONENTIAL MASS-FLOW PRESSURE-DIFFERENCE RELATION

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SUMMARY

Two-dimensional oblique shock-wave theory is used to define conditions for cancellation and reflection of shock waves from porous walls. An exponential relation between mass flow normal to the walls and pressure differential through the walls is assumed. A porosity factor is defined which uniquely determines cancellation conditions and is independent of the exponent of the mass-flow pressure-difference relation but is dependent upon the amount of wall suction. For the reflection case an approximate explicit solution for the reflected wave strength is derived and, in general, is found to be a function of the flow exponent, the amount of wall suction, and the porosity factor of the porous medium. It is pointed out that the flow across a curved three-dimensional shock wave can be related to two-dimensional flow, so that information as to the cancellation conditions for three-dimensional disturbances can be obtained from the analysis.

INTRODUCTION

Porous walls are used in transonic test sections at low supersonic speeds for the purpose of canceling or attenuating flow disturbances that ordinarily reflect from solid boundaries. These disturbances can originate from a test model or from extrinsic sources farther upstream. Two past reports, which deal theoretically with the subject, neglect two important factors which reduce their usefulness in the study of real flows. First, in general, these reports assumed a linear relation between the flow normal to the wall and the pressure difference across the wall, which experimentally is not usually the case. Second, they neglected the complicating effects of the interaction between shock waves and boundary layer at the tunnel walls so that the problem could be more easily analyzed. This, in itself, is not too serious an omission because large portions of the boundary layer in transonic wind tunnels

can be removed by porous suction, and its removal is beneficial from the standpoint of boundary-layer interaction effects and probably power requirements. However, neither report makes allowance for the application of wall suction which is required to remove the boundary layer nor other possible effects related to wall suction.

In reference 1 charts are presented which describe the wall porosity required for complete shock-wave absorption in the absence of boundary layer. Additional charts obtained by graphical methods enable the determination of reflected shock-wave strengths. For the general case, these charts are based on an assumed linear relation between pressure difference across the wall and flow normal to the wall, and in two other specific cases are based on the experimental characteristics of two porous materials having nonlinear flow relations.

In reference 2 a linear relation for the porous wall is also assumed. This report differs from reference 1 in that an approximate explicit solution for the strength of the reflected shock wave is presented as well as an equation describing shock-wave cancellation conditions.

In the present report a generalized exponential relation between pressure difference and normal flow is assumed which adequately describes porous-wall calibrations obtained experimentally at the Ames Laboratory and those presented in references 1 and 3. On this basis, conditions for oblique shock-wave cancellation and reflection are derived. All results have provisions for use of an arbitrary amount of wall suction.

SYMBOLS

a speed of sound

$\left.\begin{matrix}B\\C\end{matrix}\right\}$ constants depending on porous-wall characteristics

D suction pressure-difference ratio, $\dfrac{p_o-p_t}{p_1-p_o}$

E $1 + D$

K dimensionless porosity factor

M Mach number

$\left.\begin{matrix}m\\n\end{matrix}\right\}$ exponents depending on porous-wall characteristics, $n = \frac{1}{m} - 1$

P pressure coefficient, $\dfrac{p-p_o}{q_o}$ or $\dfrac{p-p_1}{q_1}$

p static pressure

q dynamic pressure

R perfect-gas constant

T absolute temperature

V velocity along streamline

v velocity perpendicular to free stream

v_n velocity normal to porous wall

α Mach angle, $\sin^{-1}\left(\frac{1}{M}\right)$

β $\sqrt{M^2 - 1}$

γ ratio of specific heats (1.4 for air)

δ equivalent wedge angle

θ angle of shock wave

ν angle between free stream and porous wall

ρ density

Subscripts

o upstream of initial shock wave

1 downstream of initial shock wave

2 downstream of reflected shock wave

c cancellation conditions

i initial suction

r reflection conditions

t tank enclosing test section

s suction

' conditions at same pressure difference as K_c

ANALYSIS

The purpose of this analysis is to relate the flow conditions of oblique shock waves to the flow characteristics of porous walls, based on the assumption that no boundary layer due to the tunnel walls is present. This situation will exist only if the boundary layer is removed by porous-wall suction, and therefore suction will be considered in the analysis.

It is first necessary to define the boundary condition. It is assumed that pressure drop through the porous medium follows the exponential relation

$$p - p_t = B(\rho v_n)^m \tag{1}$$

For convenience, equation (1) can be written as

$$\frac{\rho v_n}{p-p_t} = \frac{1}{B^{1/m}}(p - p_t)^{\frac{1-m}{m}} = \frac{1}{B^{1/m}}(p - p_t)^n$$

or in dimensionless form a porosity factor, K, can be defined such that

$$K = \frac{\rho v_n/\rho_o a_o}{(p-p_t)/p_o} = C\left(\frac{p-p_t}{p_o}\right)^n \tag{2}$$

By use of the perfect-gas law

$$K = \frac{\rho v_n}{p-p_t}\sqrt{\frac{RT_o}{\gamma}} \tag{2a}$$

which is a convenient form for computing K from experimental data. The constants of equations (1) and (2) are related by

$$C = \frac{1}{\rho_o a_o}\left(\frac{p_o}{B}\right)^{1/m}$$

This definition of a porosity factor, K, was first presented in reference 1 in a slightly different form. It can be interpreted as the ability of a porous medium to pass air per unit pressure difference.

Shock-Wave Cancellation

For purposes of analysis, it is convenient to begin with the case of no suction applied to the porous walls, although this is entirely hypothetical since, in the absence of suction, a boundary layer will exist at the wall. In actuality, as the suction is increased, the conditions shown in figure 1 will be more nearly approached.

The porosity factor, K, required for cancellation of an oblique shock wave can be determined by considering the flow changes through the impinging shock wave. For shock cancellation without suction (ν=0), conditions in region (1) of figure 1 must be maintained at the wall downstream of region (1), and δ_2 becomes zero. From oblique-shock-wave relations (ref. 4)

$$\frac{\rho_1}{\rho_o} = \frac{\tan \theta_1}{\tan(\theta_1-\delta_1)} \tag{3}$$

$$\frac{v_1}{a_o} = \frac{M_o \tan \delta_1}{1+\tan \delta_1 \tan \theta_1} \tag{4}$$

and

$$\frac{p_1-p_o}{p_o} = \frac{\gamma M_o^2 \tan \delta_1 \tan \theta_1}{1+\tan \delta_1 \tan \theta_1} \tag{5}$$

Therefore, the porosity factor, K, for cancellation becomes (eqs. (3), (4), and (5) substituted into the definition of K)

$$K_c = \frac{\rho v_n/\rho_o a_o}{(p-p_t)/p_o} = \frac{\rho_1 v_1/\rho_o a_o}{(p_1-p_o)/p_o}$$

or

$$K_c = \frac{\cot(\theta_1-\delta_1)}{\gamma M_o} \tag{6}$$

when $p_t = p_o$ and ν=0 (no initial suction). Since θ_1 depends only on δ_1 and M_o, K_c and $(p_1-p_o)/p_o$ are uniquely determined when δ_1 and M_o are specified. In figure 2, K_c is plotted as a function of $(p_1-p_o)/p_o$

(eqs. (6) and (5), respectively), both for constant values of deflection angle and Mach number. This type of plot is convenient because porous-wall calibrations can be superposed directly onto this grid of cancellation conditions as is shown in the figure. It is apparent that if the porosity of a particular wall is specified by equation (2), then for any free-stream Mach number, M_o, cancellation can occur at only two values of δ_1 at the most, and generally just one. This restriction can be removed by the use of wall suction.

For the case of shock-wave cancellation with wall suction, the definition of K becomes

$$K_{cs} = \frac{\rho_1 v_{n1}/\rho_o a_o}{(p_1 - p_t)/p_o} = \frac{\rho_1 V_1 \sin(\delta_1 + \nu)}{(p_1 - p_t)} \frac{p_o}{\rho_o a_o}$$

or

$$K_{cs} = \frac{\rho_1 v_1 \cos \nu + \rho_o v_{no} \cos \delta_1 \rho_1 V_1/\rho_o V_o}{(p_1 - p_o) + (p_o - p_t)} \frac{p_o}{\rho_o a_o} \tag{7}$$

Using equation (3) and the oblique-shock-wave equation

$$\frac{V_1}{V_o} = \frac{\cos \theta_1}{\cos(\theta_1 - \delta_1)} \tag{8}$$

equation (7) can be written as

$$K_{cs} = \frac{K_c \cos \nu + DK_s \sin \theta_1 \cos \delta_1/\sin(\theta_1 - \delta_1)}{1 + D} \tag{9}$$

where

$$D = \frac{p_o - p_t}{p_1 - p_o}$$

and

$$K_s = \frac{\rho_o V_o \sin \nu/\rho_o a_o}{(p_o - p_t)/p_o} = \frac{\rho_o v_{no}/\rho_o a_o}{(p_o - p_t)/p_o} \tag{10}$$

which defines the porosity factor for suction alone. In the above equations $\rho_o v_{no}$ represents the mass outflow per unit area through the porous walls for a given suction pressure differential, $p_o - p_t$. It is

seen that K_{cs} from equation (9) as well as K_c from equation (6) are point functions, and therefore cancellation can be achieved by all wall-calibration curves passing through these points, irrespective of their exponents of flow, n. Since (from eq. (2)),

$$K_{cs} = C\left(\frac{p_1-p_t}{p_o}\right)^n$$

and

$$K_s = C\left(\frac{p_o-p_t}{p_o}\right)^n$$

equation (9) can be written as

$$(1 + D)^{n+1} - \frac{\sin \theta_1 \cos \delta_1}{\sin(\theta_1-\delta_1)} D^{n+1} = \frac{C_c}{C} \cos \nu = \frac{K_c}{K'} \cos \nu \tag{11}$$

for $D \geq 0$ and $C_c/C \leq 1$. Equation (11) specifies the amount of suction pressure differential required to achieve shock-wave cancellation. It can be seen from equation (11) that for a particular wall calibration determined by C and n, cancellation can now be obtained over a range of M_o and δ_1, provided the wall suction, D, is varied in accordance with this equation. For the approximate case of δ_1 and ν approaching zero, equation (11) becomes

$$(1 + D)^{n+1} - D^{n+1} = \frac{C_c}{C} \tag{12}$$

Since equation (12) cannot be solved explicitly for D, the equation has been plotted in figure 3 with n as the parameter for the range of 0 to -0.5 (m = 1 to 2).

Shock-Wave Reflection

In the previous section the porosity factor required for shock-wave cancellation with suction was specified. It was shown that for a given porous wall, any shock wave for which $C_c/C \leq 1$ can be canceled when suction is used. For $C_c/C > 1$, impinging shock waves will be reflected as shock waves of reduced intensity. It is the purpose of this section to determine the reflected wave strength (δ_2 or p_2-p_1) as a function of the porous-wall characteristics with suction. For this case the porosity factor for reflection (K_{rs}) can be expressed as a function of conditions in region (2) of figure 1.

$$K_{rs} = \frac{\rho_2 v_{n2}/\rho_o a_o}{(p_2-p_t)/p_o} = \frac{\rho_2 V_2 \sin\left[(\delta_1-\delta_2)+\nu\right]}{(p_2-p_o)+(p_o-p_t)} \frac{p_o}{\rho_o a_o}$$

or

$$K_{rs} = \frac{\rho_2 v_2 \cos \nu + \rho_o v_{no} \cos(\delta_1-\delta_2)\rho_2 V_2/\rho_o V_o}{(p_2-p_o)+(p_o-p_t)} \frac{p_o}{\rho_o a_o} \tag{13}$$

By use of equations (3) and (8),[1] equation (13) can be written as

$$\frac{K_{rs}}{K_c} = \frac{\dfrac{K_r}{K_c}\left(1+\dfrac{p_2-p_1}{p_1-p_o}\right)\cos \nu + \dfrac{K_s}{K_c} D \dfrac{\sin \theta_2}{\sin(\theta_2-\delta_2)} \dfrac{\sin \theta_1}{\sin(\theta_1-\delta_1)} \cos(\delta_1-\delta_2)}{1 + D + \dfrac{p_2-p_1}{p_1-p_o}} \tag{14}$$

But K_r is the porosity factor for reflection without suction and can be expressed as

$$K_r = \frac{\rho_2 V_2 \sin(\delta_1-\delta_2)}{\rho_o a_o} \frac{p_o}{p_2-p_o} = K_c \frac{\rho_2 V_2 \sin(\delta_1-\delta_2)}{\rho_1 V_1 \sin \delta_1} \frac{p_1-p_o}{p_2-p_o} \tag{15}$$

or, using equations (3) and (8), equation (15) becomes

$$K_r = K_c \frac{\sin \theta_2 \sin(\delta_1-\delta_2)}{\sin(\theta_2-\delta_2)\sin \delta_1} \frac{1}{1+\dfrac{p_2-p_1}{p_1-p_o}} \tag{16}$$

Equations (14) and (16) can be combined to give

$$\frac{K_{rs}}{K_c} = \frac{\dfrac{\sin \theta_2}{\sin(\theta_2-\delta_2)}\left[\dfrac{\sin(\delta_1-\delta_2)}{\sin \delta_1}\cos \nu + \dfrac{K_s}{K_c} D \dfrac{\sin \theta_1}{\sin(\theta_1-\delta_1)} \cos(\delta_1-\delta_2)\right]}{1 + D + \dfrac{p_2-p_1}{p_1-p_o}} \tag{17}$$

[1]It is to be noted that equations (3), (4), (5), and (8) apply to region (2) when the appropriate subscripts are used.

By use of equation (5) and appropriate subscripts, equation (17) becomes

$$\frac{K_{rs}}{K_c} = \frac{\dfrac{\sin \theta_2}{\sin(\theta_2-\delta_2)}\left[\dfrac{\sin(\delta_1-\delta_2)}{\sin \delta_1} \cos \nu + \dfrac{K_s}{K_c} D \dfrac{\sin \theta_1}{\sin(\theta_1-\delta_1)} \cos(\delta_1-\delta_2)\right]}{1 + D + \dfrac{\sin \delta_2}{\sin \delta_1} \dfrac{\cos \theta_1}{\sin(\theta_1-\delta_1)} \dfrac{\sin \theta_2}{\cos(\theta_2-\delta_2)}} \tag{18}$$

Equation (18) expresses the porosity factor for reflection as a function of the reflected wave strength, δ_2 (θ_2 is a function of δ_2 and M_1). It is more useful, however, to obtain δ_2 as a function of C, n, and the initial wave strength. By equation (2)

$$\frac{K_{rs}}{K_c} = \frac{C}{C_c}\left(\frac{p_2-p_t}{p_1-p_o}\right)^n = \frac{C}{C_c}\left[\frac{(p_2-p_1) + (p_1-p_o) + (p_o-p_t)}{p_1-p_o}\right]^n$$

or

$$\frac{K_{rs}}{K_c} = \frac{C}{C_c}\left[(1 + D) + \frac{p_2-p_1}{p_1-p_o}\right]^n \tag{19}$$

Noting that

$$\frac{K_s}{K_c} = \frac{C}{C_c}\left(\frac{p_o-p_t}{p_1-p_o}\right)^n = \frac{C}{C_c} D^n \tag{20}$$

the substitution of equations (19) and (20) into equation (17) gives

$$\frac{C}{C_c} = \frac{K'}{K_c} = \frac{\dfrac{\sin \theta_2}{\sin(\theta_2-\delta_2)} \dfrac{\sin(\delta_1-\delta_2)}{\sin \delta_1} \cos \nu}{\left(1 + D + \dfrac{p_2-p_1}{p_1-p_o}\right)^{n+1} - D^{n+1} \dfrac{\sin \theta_2}{\sin(\theta_2-\delta_2)} \dfrac{\sin \theta_1}{\sin(\theta_1-\delta_1)} \cos(\delta_1-\delta_2)} \tag{21}$$

where K' is a point on the real calibration curve at the value of $(p - p_t)/p_o$ corresponding to K_c (see fig. 2). Equation (18) becomes

$$\frac{C}{C_c} = \frac{K'}{K_c} = \frac{\dfrac{\sin \theta_2}{\sin(\theta_2-\delta_2)} \dfrac{\sin(\delta_1-\delta_2)}{\sin \delta_1} \cos \nu}{\left[(1+D)+\dfrac{\sin \delta_2}{\sin \delta_1} \dfrac{\cos \theta_1}{\sin(\theta_1-\delta_1)} \dfrac{\sin \theta_2}{\cos(\theta_2-\delta_2)}\right]^{n+1} - D^{n+1} \dfrac{\sin \theta_2}{\sin(\theta_2-\delta_2)} \dfrac{\sin \theta_1}{\sin(\theta_1-\delta_1)} \cos(\delta_1-\delta_2)} \tag{22}$$

If the wall angle, ν, is sufficiently small so that $\cos \nu \approx 1$, equation (22) permits δ_2 to be determined as a function of C (or K') by graphical inversion with δ_1, n, and D as parameters. It is to be noted that C represents the calibration curve of the real wall under consideration; whereas C_c represents an imaginary calibration curve passing through any point K_c with the same exponent n as the real wall. For example, let the characteristics of the porous medium under consideration be represented by the real wall calibration curve shown in figure 2. If the incident wave, K_{cI}, has a deflection angle of 1° at $M_o = 1.20$, then the imaginary curve is the dashed line passing through point K_{cI} parallel to the real calibration curve. Points K_{cI} and K_{rI} are calculated as examples in the appendix.

To determine an approximate explicit solution for the reflected wave strength, it is first assumed that δ_2, $(p_2-p_1)/(p_1-p_o)$, and $\Delta\theta$ are small compared to unity where $\theta_2 = \alpha_1 + \Delta\theta$. Then, by use of the binomial series, neglecting squared terms of small quantities, and letting $E = 1 + D$, the pressure term of equation (21) becomes

$$\left(E + \frac{p_2-p_1}{p_1-p_o}\right)^{n+1} = E^{n+1}\left(1 + \frac{n+1}{E} \frac{p_2-p_1}{p_1-p_o} + \ldots\right)$$

so that equation (21) becomes

$$\frac{K'}{K_c} E^{n+1}\left(1 + \frac{n+1}{E} \frac{p_2-p_1}{p_1-p_o}\right) = (1-\delta_2 \cot \delta_1 + \beta_1\delta_2)\cos \nu +$$

$$D^{n+1} \frac{K'}{K_c} \frac{\sin \theta_1 \cos \delta_1}{\sin(\theta_1-\delta_1)} \left[1 + \delta_2(\tan \delta_1 + \beta_1)\right] \tag{23}$$

The pressure term can be approximated by

$$\frac{p_2-p_1}{p_1-p_0} = \frac{\sin \delta_2}{\sin \delta_1} \frac{\cos \theta_1}{\sin(\theta_1-\delta_1)} \frac{\sin \theta_2}{\cos(\theta_2-\delta_2)} = \frac{\delta_2 \cos \theta_1}{\beta_1 \sin(\theta_1-\delta_1) \sin \delta_1} \tag{24}$$

Inserting equation (24) into equation (23) and solving for δ_2, we obtain

$$\delta_2 = \frac{\beta_1 \sin(\theta_1-\delta_1)\sin \delta_1 \left[\cos \nu - \frac{K'}{K_c}\left(E^{n+1} - D^{n+1} \frac{\sin \theta_1}{\sin(\theta_1-\delta_1)} \cos \delta_1\right)\right]}{\beta_1 \cos \nu \sin(\theta_1-\delta_1)(\cos \delta_1-\beta_1 \sin \delta_1) + \frac{K'}{K_c}(n+1)E^n \cos \theta_1 \left[1-\beta_1 \frac{D^{n+1} \tan \theta_1 \sin \delta_1}{E^n(n+1)}(\sin \delta_1+\beta_1 \cos \delta_1)\right]} \tag{25}$$

Equation (25), therefore, is an approximate solution for the reflected wave strength (δ_2) when it is small compared to the initial wave. As a further approximation, δ_1 can be assumed to approach zero and $\cos \nu = 1$. Then equation (25) becomes

$$\delta_2 = \frac{\delta_1 \left[1 - \frac{K'}{K_c}\left(E^{n+1} - D^{n+1}\right)\right]}{1 + \frac{K'}{K_c} E^n(n+1)} \tag{26}$$

The pressure coefficient of the reflected wave can be written to a first approximation as (ref. 4, p. 134)

$$P_r = \frac{p_2-p_1}{q_0} = \frac{q_1}{q_0} \frac{p_2-p_1}{q_1} = \frac{\rho_1 V_1^{\,2}}{\rho_0 V_0^{\,2}} \frac{2\delta_2}{\beta_1}$$

or by use of equations (3) and (8)

$$P_r = \frac{2 \sin 2\theta_1}{\beta_1 \sin 2(\theta_1-\delta_1)} \delta_2 \tag{27}$$

Equations (25) and (27) can be combined to give

$$P_r = \frac{\sin 2\theta_1 \sin \delta_1 \left[\cos \nu - \frac{K'}{K_c}\left(E^{n+1} - D^{n+1} \frac{\sin \theta_1}{\sin(\theta_1 - \delta_1)} \cos \delta_1\right)\right]}{\cos(\theta_1 - \delta_1)\left\{\beta_1 \cos \nu \sin(\theta_1 - \delta_1)(\cos \delta_1 - \beta_1 \sin \delta_1) + \frac{K'}{K_c}(n+1)E^n \cos \theta_1 \left[1 - \beta_1 \frac{D^{n+1} \tan \theta_1 \sin \delta_1}{E^n(n+1)}(\sin \delta_1 + \beta_1 \cos \delta_1)\right]\right\}} \tag{28}$$

When $\delta_1 \longrightarrow 0$ and $\cos \nu = 1$ as for equation (26),

$$P_r = \frac{2\delta_1\left[1 - \frac{K'}{K_c}\left(E^{n+1} - D^{n+1}\right)\right]}{\beta_o\left[1 + \frac{K'}{K_c} E^n(n+1)\right]} \tag{29}$$

With no suction ($D = 0$), $(K_c - K')/K_c$ is of the same order as P_r or δ_2, so that equations (26) and (29) become

$$\delta_2 = \frac{\delta_1 (K_c - K')/K_c}{n + 2} = \frac{\delta_1 \Delta K/K_c}{n + 2} \tag{30}$$

and

$$P_r = \frac{2\delta_1 \Delta K/K_c}{\beta_o(n+2)} \tag{31}$$

In equations (25) to (31), a positive value of δ_2 or P_r represents a compressive reflection and a negative value an expansive reflection. A typical value of ΔK is indicated in figure 2.

In the foregoing equations allowance has been made for the use of suction. Since the amount of air to be removed $(\rho_o v_{no})$ by suction alone is a predetermined factor, the suction pressure differential p_o-p_t and, therefore, D may be obtained by using equation (2) in the form

$$\frac{p_o-p_t}{p_o} = \left(\frac{1}{C}\frac{\rho_o v_{no}}{\rho_o a_o}\right)^{\frac{1}{n+1}} \tag{32}$$

DISCUSSION

For the practical application of the equations developed in this paper, it is necessary first to obtain experimental data on the characteristics of the porous wall under consideration; that is, determine C and n of equation (2). On the basis of reference 5, C and n would be expected to be a function of the free-stream velocity and secondary factors such as Reynolds number. Since T_o is a function of free-stream Mach number and total temperature, K and hence C and n are easily determined by the remaining measurable quantities of equation (2a).

In the most general form without any simplifying assumptions, equation (22) relates the porous-wall characteristics (K, C, and n) to the reflected wave strength (δ_2). As previously stated, δ_2 can be determined, for a given wall calibration and suction, by graphical inversion when $\cos \nu = 1$. Equations (26) and (29) give explicit solutions for the reflected wave strengths in their most approximate form with suction. When δ_2 in equation (22) is made equal to zero, equation (11) is obtained, which relates the porous-wall characteristics and incident wave strength, p_1-p_o, to the suction for cancellation. By definition, the porosity factor through which a wall calibration curve must pass for shock cancellation with wall suction is given by equation (9), where K_c is related to the incident shock wave by equation (6).

In reference 3 the effect of suction on the sign and magnitude of reflected disturbances is discussed, and it is pointed out that suction tends to make a porous wall appear more closed. This effect can be observed in figure 2, since K for the calibration curves decreases with increasing pressure difference. In reference 3 it was also indicated that, with a single porous wall, a selected amount of suction can produce cancellation over a range of incident waves and Mach numbers which otherwise would result in reflections. As an example, let K_{cII} in figure 2 represent a point for cancellation of an incident wave without suction. According to equation (30) or (31), an expansive reflection will result in proportion to $(K_{cII}-K'_{II})/K_{cII}$, which is negative. Then equation (12) allows an approximate computation of D and hence (p_o-p_t), which is the wall suction required to cancel the reflected wave. In the more general case, an initial amount of suction, p_o-p_{ti}, may be

used which does not achieve complete cancellation of the reflected wave, as indicated in the figure by K_{csi}. Then the additional amount of suction required to make this point coincide with K_{csII} is $(p_{ti}-p_t)$. It is to be noted that the point K_{csi} may be above the real calibration curve, in which case the suction must be reduced in order to achieve cancellation. In summary, suction translates points on the cancellation grid of figure 2 to the right and up, which results in a more solid effect of the porous medium. Calculations for these examples are given in the appendix, example II.

Since the analysis of this report was based on two-dimensional relations, the question arises as to the possibility of applying these relations to three-dimensional flows. In references 6 and 7 it was shown that for a given pressure ratio and bow-wave angle, there is both a wedge and a cone that can produce these conditions. This simply means that the flow immediately behind a conical bow shock is essentially two-dimensional. If this result is assumed to apply to a curved three-dimensional bow wave, then it is possible to represent this three-dimensional shock disturbance by an equivalent two-dimensional disturbance at the tangent point of the waves. This is shown schematically in the lower left of figure 2. In practice, this procedure amounts to determining the wave strength near the wall of a three-dimensional body either by direct measurement or the method of characteristics (when applicable), and then selecting a two-dimensional deflection angle to correspond to the three-dimensional pressure rise. A result of this procedure by direct measurement is shown in figure 2 for a fineness ratio 12 body of revolution (RM 12) which amounted to 0.34-percent blocked area in a 5- by 5-inch transonic test section. This curve was obtained by measuring the pressure changes, $(p_1-p_o)/p_o$, through the bow wave near the wall and plotting these values at the appropriate Mach numbers in figure 2. This is equivalent to computing K_c from equation (6) and plotting it at the proper value of $(p_1-p_o)/p_o$. This procedure when applied to more complex bodies at various angles of attack can provide information as to the range of wall porosities that is needed for approximate interference-free testing of three-dimensional models.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., May 7, 1954

APPENDIX

NUMERICAL EXAMPLES

Example I.- Calculation of Coordinates for Points K_{cI} and K_{rI} in Figure 2

Given:

$M_o = 1.20$

$\delta_1 = 1.0^o$

$\theta_1 = 58.6^o$ (from oblique shock relations, ref. 4)

No suction ($D = 0$)

Wall calibration, $K = 0.103 \left(\frac{p-p_t}{p_o}\right)^{-0.40}$

To find K_{cI}

From equation (6)

$$K_{cI} = \frac{\cot(\theta_1-\delta_1)}{\gamma M_o} = \frac{\cot(58.6-1.0)}{1.400 \times 1.200} = 0.378$$

From equation (5)

$$\frac{p_1-p_o}{p_o} = \frac{\gamma M_o{}^2 \tan \delta_1 \tan \theta_1}{1+\tan \delta_1 \tan \theta_1} = \frac{1.400 \times 1.440 \times 0.01745 \times 1.638}{1 + 0.01745 \times 1.638} = 0.0560$$

Using the wall calibration

$$K'_I = 0.103 \left(\frac{p-p_t}{p_o}\right)^{-0.40} = 0.103(0.0560)^{-0.40} = 0.326$$

To find K_{rI}:

From oblique shock relations, $M_1 = 1.160$. Then, for $D = 0$, equation (28)

becomes

$$P_r = \frac{\sin 2\theta_1 \sin \delta_1 \left[(K_c - K')/K_c\right]}{\cos(\theta_1 - \delta_1)\left[\beta_1 \sin(\theta_1 - \delta_1)(\cos \delta_1 - \beta_1 \sin \delta_1) + \frac{K'}{K_c}(n+1)\cos \theta_1\right]}$$

$$= \frac{0.889 \times 0.0175\left[(0.378 - 0.326)/0.378\right]}{0.536\left[0.588 \times 0.844(1.00 - 0.588 \times 0.0175) + 0.862 \times 1.60 \times 0.521\right]} = 0.00329$$

and

$$\frac{p_2 - p_1}{p_o} = \frac{\gamma}{2} M_o^2 P_r = 0.700 \times 1.44 \times 0.00329 = 0.0033$$

Therefore,

$$\frac{p_2 - p_o}{p_o} = \frac{p_1 - p_o}{p_o} + \frac{p_2 - p_1}{p_o} = 0.0560 + 0.0033 = 0.0593$$

and

$$K_{rI} = 0.103\left(\frac{p - p_t}{p_o}\right)^{-0.40} = 0.103(0.0593)^{-0.40} = 0.319$$

If suction were applied to the porous walls for the above example, the reflection strength, P_r, would become greater.

Example II.- Calculation of Suction Pressure Differential Required to Achieve Cancellation

Given:

$M_o = 1.10$

$\delta_1 = 1/2^\circ$

$$\left.\begin{array}{l} K_{cII} = 0.278 \\ (p_1 - p_o)/p_o = 0.0347 \end{array}\right\} \text{computed similar to example I}$$

Wall calibration, $K = 0.103\left(\frac{p - p_t}{p_o}\right)^{-0.40}$

Since δ_1 is small, equation (12) (fig. 3) can be used to calculate the suction pressure differential, $(p_o-p_t)/p_o$, required to attain cancellation. It is first necessary to determine C_c/C which depends on K_{II}'. From the wall calibration

$$K_{II}' = 0.103(0.0347)^{-0.40} = 0.395$$

Then

$$\frac{C_c}{C} = \frac{K_c}{K'} = \frac{0.278}{0.395} = 0.704$$

and from figure 3

$$D = 0.258$$

Therefore, the suction pressure differential required is

$$\frac{p_o-p_t}{p_o} = \frac{p_1-p_o}{p_o} D = 0.0347 \times 0.258 = 0.0090$$

Since

$$\frac{p_1-p_t}{p_o} = \frac{p_1-p_o}{p_o} + \frac{p_o-p_t}{p_o} = 0.0347 + 0.0090 = 0.0437$$

$$K_{csII} = 0.103\,(0.0437)^{-0.40} = 0.360$$

If an initial suction pressure differential, which does not produce cancellation, is imposed on the test-section walls, the additional suction required, $(p_{ti}-p_t)/p_o$, can be determined as follows:

Let $(p_o-p_{ti})/p_o = 0.0050$ be the initial suction. Then

$$\frac{p_{ti}-p_t}{p_o} = \frac{p_o-p_t}{p_o} - \frac{p_o-p_{ti}}{p_o} = 0.0090 - 0.0050 = 0.0040$$

The point K_{csi}, corresponding to the initial suction, can be computed as follows:

From the wall calibration

$$K_{si} = 0.103(0.0050)^{-0.40} = 0.858$$

and

$$D_i = \frac{0.0050}{0.0347} = 0.144$$

Using equation (9) for $\delta_1 \longrightarrow 0$

$$K_{csi} = \frac{K_{cII} + D_i K_{si}}{1 + D_i} = \frac{0.278 + 0.144 \times 0.858}{1 + 0.144} = 0.353$$

and

$$\frac{p_1 - p_{ti}}{p_o} = \frac{p_1 - p_o}{p_o} + \frac{p_o - p_{ti}}{p_o} = 0.0347 + 0.0050 = 0.0397$$

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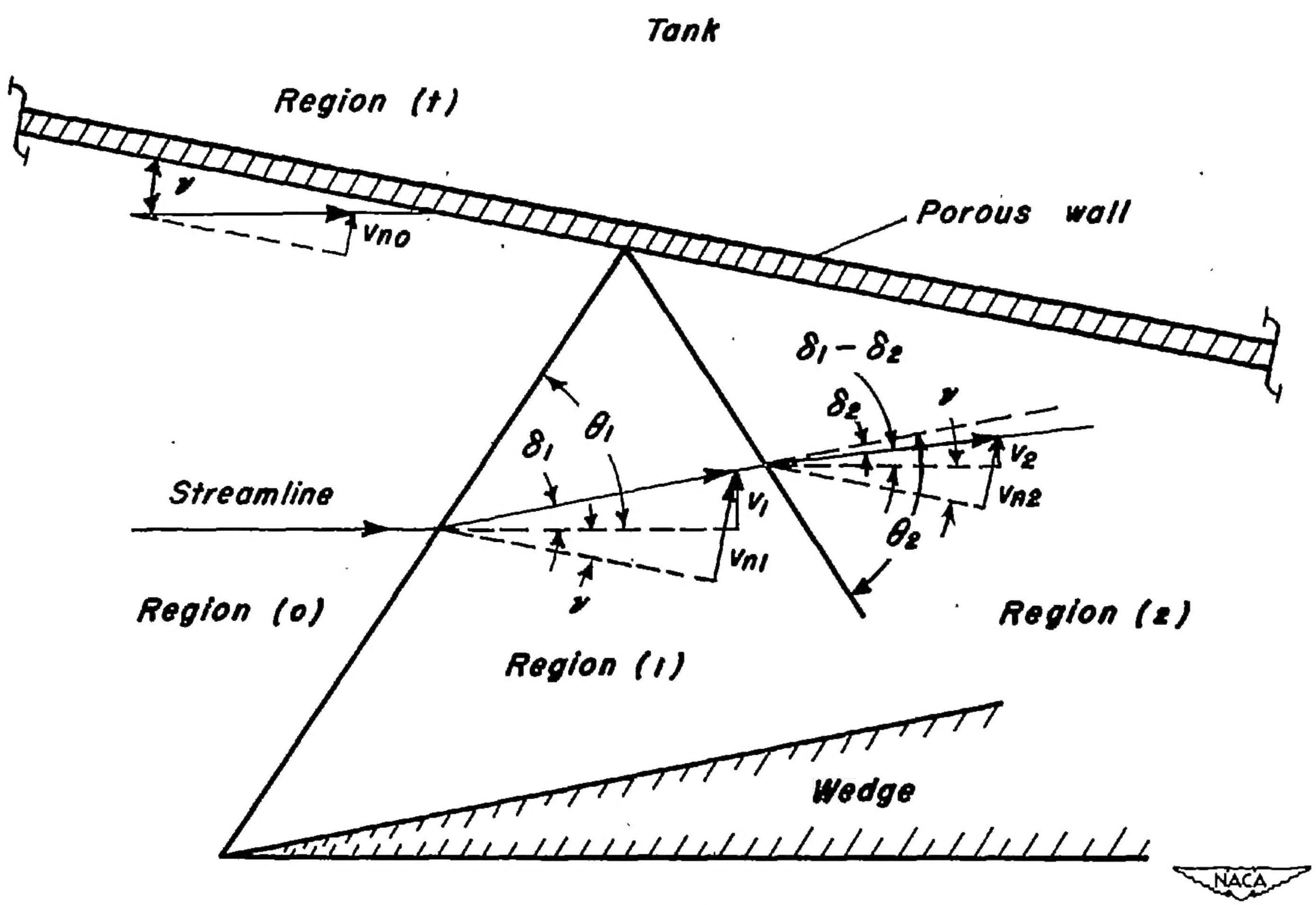

Figure 1.- Conditions for shock-wave reflection from a porous wall without boundary layer.

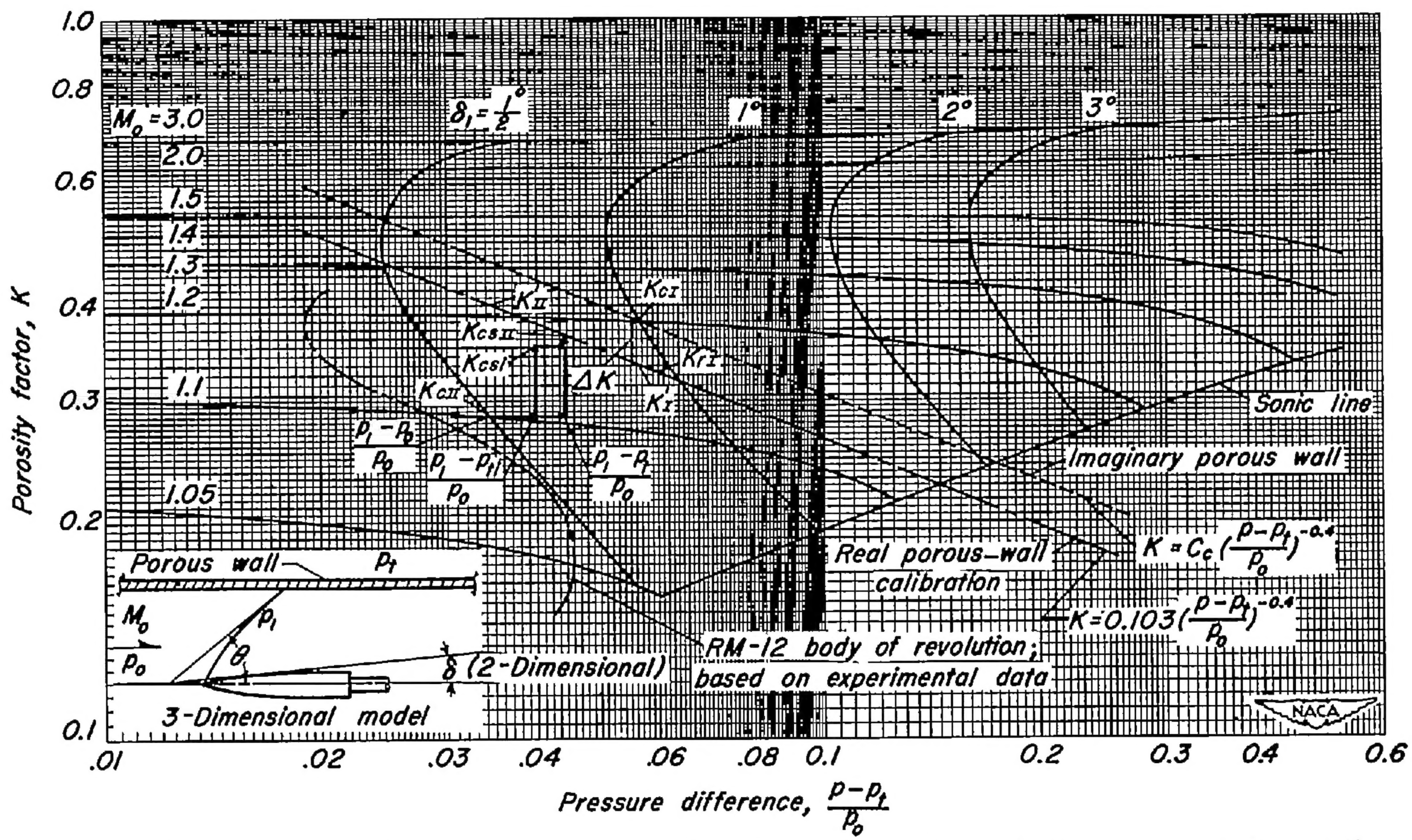

Figure 2.- Theoretical porosity factor for cancellation of two-dimensional shock waves and its relation to a porous-wall calibration of the form $K = C(\Delta p/p_o)^n$.

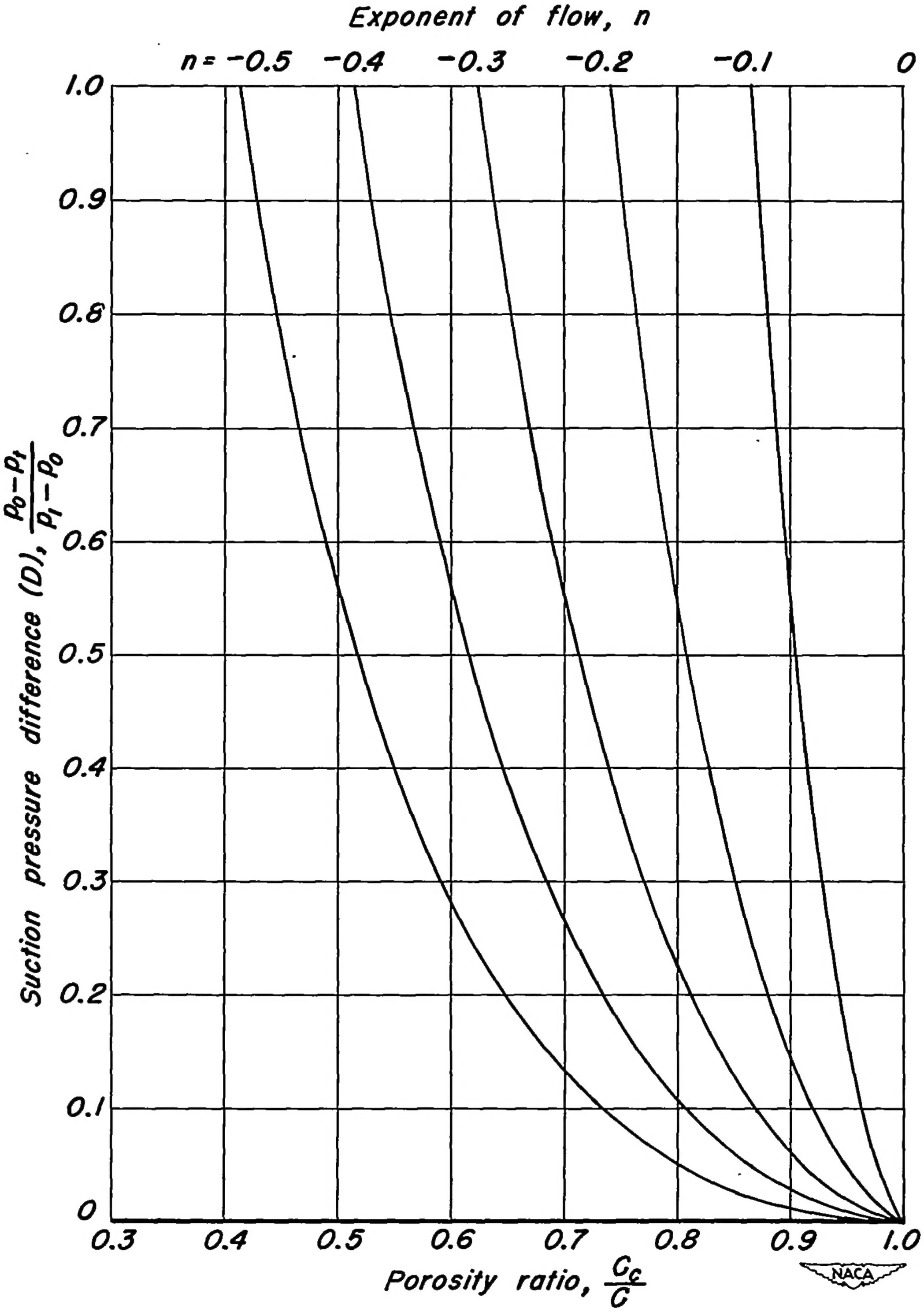

Figure 3.– Approximate relation of suction pressure difference to porosity constant ratio for shock-wave cancellation.

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